

# Mathematics!



## **A Story of Units Parent Handbook**

**Grade 3  
Module 1**

## Grade 3 • Module 1

### Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

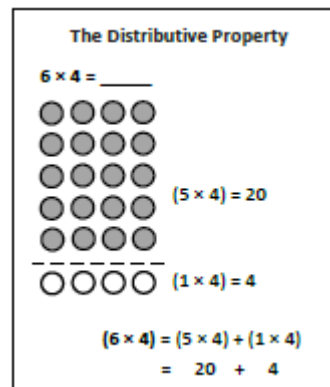
#### OVERVIEW

This 25-day module begins the year by building on students' fluency with addition and knowledge of arrays. Topic A initially uses repeated addition to find the total from a number of equal groups. As students notice patterns, they let go of longer addition sentences in favor of more efficient multiplication facts. Lessons in Topic A move students toward understanding familiar repeated addition from Grade 2 in the form of array models, which become a cornerstone of the module. Students use the language of multiplication as they understand what factors are and differentiate between the size of groups and the number of groups within a given context. In this module the factors 2, 3, 4, 5, and 10 provide an entry point for moving into more difficult factors in later modules.

Study of factors links Topics A and B; Topic B extends the study to division. Students understand division as an unknown factor problem, and relate the meaning of unknown factors to either the number or the size of groups. By the end of Topic B students are aware of a fundamental connection between multiplication and division that sets the foundation for the rest of the module.

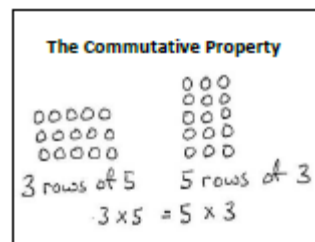
In Topic C, students use the array model and familiar skip-counting strategies to solidify their understanding of multiplication and practice related facts of 2 and 3. They become fluent enough with arithmetic patterns to “add” or “subtract” groups from known products to solve more complex multiplication problems. They apply their skills to word problems using drawings and equations with a symbol to find the unknown factor. This culminates in students using arrays to model the distributive property as they decompose units to multiply.

In Topic D students model, write and solve partitive and measurement division problems with 2 and 3. Consistent skip-counting strategies and the continued use of array models are pathways for students to naturally relate multiplication and division. Modeling advances as students use tape diagrams to represent multiplication and division. A final lesson in this topic solidifies a growing understanding of the relationship between operations.



Topic E shifts students from simple understanding to analyzing the relationship between multiplication and division. Practice of both operations is combined—this time using units of 4—and a lesson is explicitly dedicated to modeling the connection between them. Skip-counting, the distributive property, arrays, number bonds and tape diagrams are tools for both operations. A final lesson invites students to explore their work with arrays and related facts through the lens of the commutative property as it relates to multiplication.

Topic F introduces the factors 5 and 10, familiar from skip-counting in Grade 2. Students apply the multiplication and division strategies they have used to mixed practice with all of the factors included in Module 1. Students model relationships between factors, analyzing the arithmetic patterns that emerge to compose and decompose numbers as they further explore the relationship between multiplication and division.



In the final lesson of the module, students apply the tools, representations, and concepts they have learned to problem-solving with multi-step word problems using all four operations. They demonstrate the flexibility of their thinking as they assess the reasonableness of their answers for a variety of problem types.

The mid-module assessment follows Topic C. The end-of-module assessment follows Topic F.

# Terminology

## New or Recently Introduced Terms

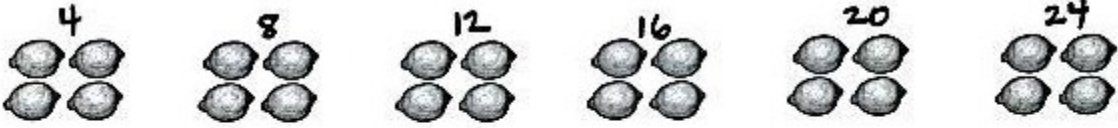
- Array (a set of numbers or objects that follow a specific pattern, a matrix)
- Column (e.g., in an array)
- Commutative Property/Commutative (e.g., rotate a rectangular array 90 degrees to demonstrate that factors in a multiplication sentence can switch places)
- Equal groups (with reference to multiplication and division; one factor is the number of objects in a group and the other is a multiplier that indicates the number of groups)
- Equation (a statement that 2 expressions are equal. E.g.,  $3 \times 4 = 12$ )
- Distribute (with reference to the Distributive Property; e.g. In  $12 \times 3 = (10 \times 3) + (2 \times 3)$  the 3 is multiplier for each part of the decomposition)
- Divide/division (partitioning a total into equal groups to show how many equal groups add up to a specific number. E.g.,  $15 \div 5 = 3$ )
- Fact (used to refer to multiplication facts, e.g.,  $3 \times 2$ )
- Factors (i.e., numbers that are multiplied to obtain a product)
- Multiplication/multiply (an operation showing how many times a number is added to itself e.g.,  $5 \times 3 = 15$ )
- Number of groups (factor in a multiplication problem that refers to the total equal groups)
- Parentheses (e.g., ( ) used around a fact or numbers within an equation)
- Quotient (the answer when one number is divided by another)
- Rotate (turn, used with reference to turning arrays 90 degrees)
- Row/column (in reference to rectangular arrays)
- Size of groups (factor in a multiplication problem that refers to how many in a group)
- Unit (i.e., one segment of a partitioned tape diagram)
- Unknown (i.e., the “missing” factor or quantity in multiplication or division)

## Familiar Terms and Symbols

- Add 1 unit, subtract 1 unit (add or subtract a single unit of two, ten, etc.)
- Number bond (shows part-part-whole relationship, shown at right)
- Number sentence (similar to an equation, but not necessarily having equal sides.)
- Ones, twos, threes, etc. (units of one, two, or three)
- Repeated addition (adding equal groups together, e.g.,  $2 + 2 + 2 + 2$ )
- Tape Diagram (a method for modeling problems)
- Value (how much)

## Lesson 1

Objective: Understand *equal groups* of as multiplication.



d.  $4 + 4 + 4 + 4 + 4 + 4 = 24$   
6 groups of  $4 = 24$   
 $6 \times 4 = 24$

The picture below shows 2 groups of apples. Does the picture below show  $2 \times 3$ ? Explain why or why not.



No, this picture does not show  $2 \times 3$ . There are 2 groups of apples, but they are not equal groups. It shows 1 group of 3 and 1 group of 2. You can also tell because there are 5 apples, not 6.

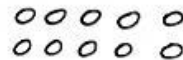
## Lesson 2

Objective: Relate multiplication to the array model.

5. The dots below show 2 groups of 5.



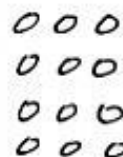
a. Redraw the circles as an array that shows 2 rows of 5.



b. Compare the drawing to your array. Write at least 1 reason why they are the same and 1 reason why they are different.

They are the same because they have the same amount of circles - 10 circles. They are different because the array is in rows and the other ones are just in whatever place. The array is easier to see.

Emma collects rocks. She arranges them in 4 rows of 3. Draw Emma's array to show how many rocks she has altogether. Then write a multiplication sentence to describe the array.



$4 \times 3 = 12$   
Emma has 12 rocks.

# Lesson 3

Objective: Interpret the meaning of factors – the size of the group or the number of groups.

a. Write a multiplication sentence for the array shown below.

$$\begin{array}{c} \text{XXX} \\ \text{XXX} \\ \text{XXX} \\ \text{XXX} \end{array} \quad 4 \times 3 = 12$$

b. Draw a number bond for the array where each part represents the amount in one row.

There are 4 oranges in each row. How many oranges are there in 3 rows?

a) Number of rows: 3      Size of each row: 4

b) 3  $\times$  4 = 12

c) There are 12 oranges altogether.

# Lesson 4

Objective: Understand the meaning of the unknown as the size of the group in division.

1. Divide 14 flowers into 2 equal groups. There are 7 flowers in each group.

2. Divide 28 books into 4 equal groups. There are 7 books in each group.

3. Audrina has 24 colored pencils. She puts them in 4 equal groups. How many colored pencils are in each group?

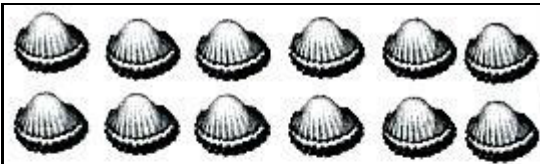
There are 6 colored pencils in each group.  
 $24 \div 4 = \underline{6}$

4. Charlie picks 20 apples. He divides them equally between 5 baskets. Draw the apples in each basket.

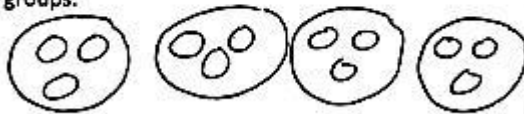
There are 4 apples in each basket.  
 $20 \div \underline{5} = \underline{4}$

# Lesson 5

Objective: Understand the meaning of the unknown as the number of groups in division.

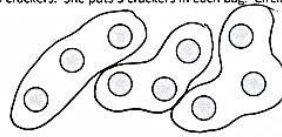


Divide the shells to show  $12 \div 3 = \underline{4}$  where the unknown represents the number of groups.

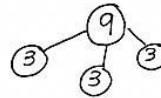


How many groups are there? 4

Rachel has 9 crackers. She puts 3 crackers in each bag. Circle the crackers to show Rachel's bags.



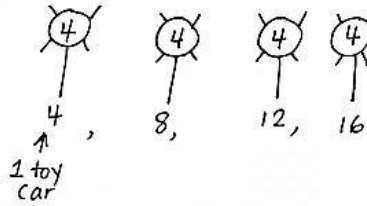
- a. Write a division sentence where the answer represents the number of Rachel's bags.  $9 \div 3 = 3$
- b. Draw a number bond to show Rachel's crackers.



Rachel can make 3 bags with 9 crackers.

Jameisha has 16 wheels to make toy cars. She uses 4 wheels for 1 car.

- a. Use a count-by to find the number of cars Jameisha can build. Make a drawing to match your counting.



- b. Write a division sentence to represent the problem.

$16 \div 4 = 4$

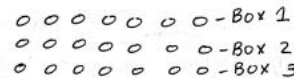
Jameisha can make 4 toy cars with 16 wheels.

# Lesson 6

Objective: Interpret the unknown in division using the array model.

- 4. Deena makes 21 jars of tomato sauce on her farm. She puts 7 jars in each box to sell at the supermarket. How many boxes does Deena need?

$21 \div 7 = \underline{3}$   
 $\underline{3} \times 7 = 21$



What is the meaning of the unknown factor and quotient? It represents the number of groups.

- 5. The teacher gives the problem  $4 \times \underline{3} = 12$ . Charlie finds the answer by writing and solving  $12 \div 4 = \underline{3}$ . Explain why Charlie's method works.



Charlie's method works because in both problems we have 4 groups of 3 and a total of 12. The quotient in a division problem is like finding an unknown factor in a multiplication problem.


- 6. The blanks in problem 5 represent the size of the groups. Draw an array to represent the number sentences.



## Lesson 7


Objective: Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

1. a) Count by 2 six times.  
2, 4, 6, 8, 10, 12

b) Draw an array that matches your count-by.  


c) Write a multiplication sentence that represents the total number of objects in your array.  
6 x 2 = 12

2. a) Count by 6 two times.  
6, 12

b) Draw an array that matches your count-by.  


c) Write a multiplication sentence that represents the total number of objects in your array.  
2 x 6 = 12

3. a) Compare your work in problems 1 and 2. Turn your paper as you study the arrays to look at them in different ways.  
*It's the same array! The array in problem 1 just gets turned on its side in problem 2.*

b) Why are the factors in your multiplication sentences in a different order?  
*Problem 1 you read as 6 groups and 2 in each group. Problem 2 you read as 2 groups with 6 in each group. So the problems are the same, but the numbers are arranged in a different order.*

4. Count by the unit (the number in word form) the number of times indicated. Write the multiplication sentence that matches your count by. The first one is done for you.

a) 6 twos: <u>6 x 2 = 12</u>	d) 2 sevens: <u>2 x 7 = 14</u>	Bonus Questions:
b) 2 sixes: <u>2 x 6 = 12</u>	e) 9 twos: <u>9 x 2 = 18</u>	g) 11 twos: <u>11 x 2 = 22</u>
c) 7 twos: <u>7 x 2 = 14</u>	f) 2 nines: <u>2 x 9 = 18</u>	h) 2 twelves: <u>2 x 12 = 24</u>

## Lesson 8

Objective: Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

Isaac picks 3 tangerines from his tree every day for 7 days.

a. Use circles to draw an array that represents the tangerines Isaac picks.



b. How many tangerines does Isaac pick in 7 days? Write and solve a multiplication sentence.

7 x 3 = 21 Isaac picks 21 tangerines in 7 days.

c. Isaac decides to pick 3 tangerines every day for 3 more days. Draw 'x's to show the new tangerines on the array in part A.

d. Write and solve a multiplication sentence to find the total number of tangerines Isaac picks.

10 x 3 = 30 He picks 30 tangerines altogether.

a) 2 threes: 2 x 3 = 6

b) 3 twos: 3 x 2 = 6

c) 3 fours: 3 x 4 = 12



# Lesson 9

Objective: Find related multiplication facts by adding and subtracting equal groups in array models.

1. The team organizes soccer balls into 2 rows of 5. The coach adds 3 rows of 5 soccer balls. Complete the number sentences to describe the total array.

a.  $(5 + 5) + (5 + 5 + 5) = 25$   
 b. 2 fives + 3 fives = 5 fives  
 c.  $5 \times 5 = 25$

2.  $7 \times 2 = 14$

$5 \times 2 = 10$   
 $2 \times 2 = 4$

$10 + 4 = 14$   
 $7 \times 2 = 14$

3.  $9 \times 2 = 18$

$10 \times 2 = 20$   
 $1 \times 2 = 2$

$20 - 2 = 18$   
 $9 \times 2 = 18$

# Lesson 10

Objective: Model the distributive property with arrays to decompose units as a strategy to multiply.

2.  $8 \times 3 = (4 \times 3) + (4 \times 3) = 24$

$4 \times 3 = 12$   
 $4 \times 3 = 12$

$(4 \times 3) + (4 \times 3) = 12 + 12$   
 $8 \times 3 = 24$

3. Ruby is making a photo album. She puts 3 pictures in each row.

a) Use the multiplication sentences on the left. Draw arrays to show the photos on the upper and lower parts of Ruby's album page.

$2 \times 3 = 6$   
 $3 \times 3 = 9$

b) Ruby calculates the total number of pictures as shown below. Use the array you drew to help explain her calculation.

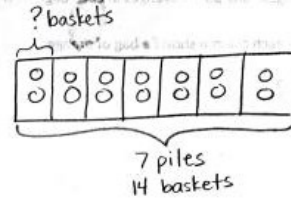
$5 \times 3 = 6 + 9 = 15$

The whole array shows 5 rows times 3 columns. So  $5 \times 3$ . Then maybe Ruby didn't know the answer to  $5 \times 3$  so she broke it into 2 smaller facts.  $2 \times 3$ , which is 6, and  $3 \times 3$ , which is 9. So she did  $6 + 9$ . Because  $5 \times 3 = 6 + 9$ . Then if you do  $5 \times 3$  it's 15. And  $6 + 9$  is 15. So  $5 \times 3 = 6 + 9 = 15$ .

# Lesson 11

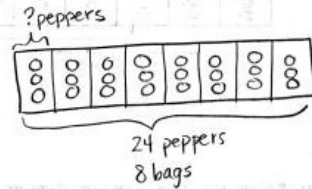
Objective: Model division as the unknown factor in multiplication using arrays and tape diagrams.

3. Fourteen shopping baskets are stacked equally in 7 piles. How many baskets are in each pile? Model the problem with both an array and a labeled tape diagram. Show each column as the number of baskets in each pile.



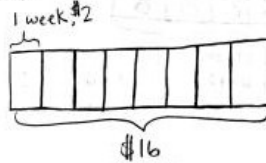
$14 \div 7 = 2$   
There are 2 shopping baskets in each pile.

4. In the back of the store, Mr. Prescott packs 24 bell peppers equally into 8 bags. How many bell peppers are in each bag? Model the problem with both an array and a labeled tape diagram. Show each column as the number of bell peppers in each bag.



$24 \div 8 = 3$   
There are 3 peppers in each bag.

5. Olga saves \$2 a week to buy a toy car. The car costs \$16. How many weeks will it take her to save enough to buy the toy?

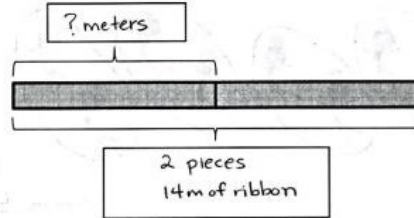


$\$16 \div \$2 = 8$   
It will take Olga 8 weeks to save up.

# Lesson 12

Objective: Interpret the quotient as the number of groups or the number of objects in each group using units of 2.

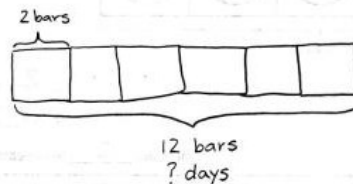
4. Laina buys 14 meters of ribbon. She cuts her ribbon into 2 equal pieces. How many meters long is each piece? Label the tape diagram to represent the problem, including the unknown.



$14 \div 2 = 7$

Each piece is 7 meters long.

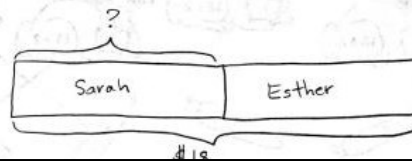
5. Roy eats 2 cereal bars every morning. Each box has a total of 12 bars. How many days will it take Roy to finish 1 box?



$12 \div 2 = 6$

It will take 6 days to finish 1 box.

6. Sarah and Esther equally share the cost of a present. The present costs \$18. How much does Sarah pay?

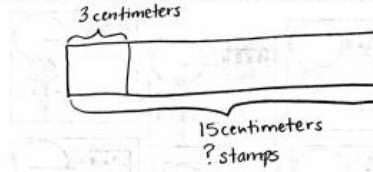


$18 \div 2 = 9$   
Sarah pays \$9.

## Lesson 13

Objective: Interpret the quotient as the number of groups or the number of objects in each group using units of 3.

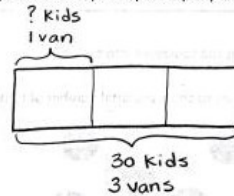
3. Camille buys a sheet of stamps that measures 15 centimeters long. Each stamp is 3 centimeters long. How many stamps does Camille buy? Draw and label a tape diagram to solve.



$$15 \div 3 = 5$$

Camille buys 5 stamps.

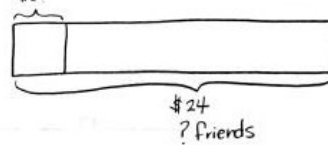
4. Thirty third graders go on a field trip. They are equally divided into 3 vans. How many students are in each van?



$$30 \div 3 = 10$$

There are 10 students in each van.

5. Some friends spend \$24 altogether on frozen yogurt. Each person pays \$3. How many people buy frozen yogurt?



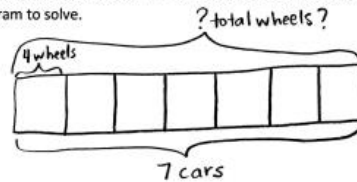
$$\$24 \div \$3 = 8$$

8 friends buy frozen yogurt.

## Lesson 14

Objective: Skip-Count objects in models to build fluency with multiplication facts using units of 4.

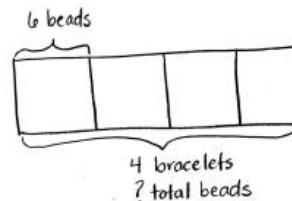
2. Mr. Schmidt replaces each of the 4 wheels on 7 cars. How many wheels does he replace? Draw and label a tape diagram to solve.



$$7 \times 4 = 28$$

Mr. Schmidt replaces 28 wheels altogether.

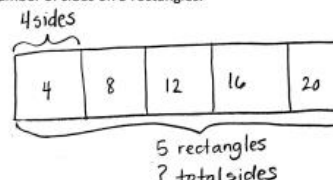
3. Trina makes 4 bracelets. Each bracelet has 6 beads. Draw and label a tape diagram to show the total number of beads Trina uses.



$$4 \times 6 = 24$$

Trina uses 24 beads total.

4. Find the total number of sides on 5 rectangles.



$$5 \times 4 = 20$$

There are 20 sides on 5 rectangles.

# Lesson 15

Objective: Relate arrays to tape diagrams to model the commutative property of multiplication.

4  
 $4 \times 3 = 12$

3  
 $3 \times 4 = 12$

12

12

••••  
 ••••  
 ••••

2. Draw and label 2 tape diagrams to model how the statement in the box is true.  $4 \times 6 = 6 \times 4$

6  
 $4 \times 6 = 24$

4  
 $6 \times 4 = 24$

$4 \times 6 = 6 \times 4$

3. Grace picks 4 flowers from her garden. Each flower has 8 petals. Draw and label a tape diagram to show how many petals there are in total.

8 petals

4 flowers  
 ? petals

$4 \times 8 = 32$

There are 32 petals on Grace's 4 flowers.

4. Michael counts 8 chairs in his dining room. Each chair has 4 legs. How many chair legs are there altogether?

4 legs

8 chairs  
 ? legs

$4 \times 8 = 32$

There are 32 chair legs altogether.

# Lesson 16

Objective: Use the distributive property as a strategy to find related multiplication facts.

d)  $9 \times 4 = 36$

$(5 \times 4) = 20$

$(4 \times 4) = 16$

$(9 \times 4) = (5 \times 4) + (4 \times 4)$

$= 20 + 16$

$= 36$

3. Nolan draws the array below to find the answer to the multiplication fact  $4 \times 10$ . He says, " $4 \times 10$  is just double  $4 \times 5$ !" Explain Nolan's strategy.

$4 \times 5 = 20$

$4 \times 5 = 20$





















$20 + 20 = 40$

$4 \times 5 = 20$  and  $20 + 20 = 40$ . If I know  $4 \times 5 = 20$  then I can double the answer and I will know  $4 \times 10 = 40$  because 10 is the double of 5.

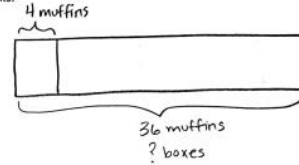
# Lesson 17

Objective: Model the relationship between multiplication and division.

1. Use the array to complete the related number sentences.

		$1 \times 4 = 4$	$4 \div 4 = 1$
		$2 \times 4 = 8$	$8 \div 4 = 2$
		$3 \times 4 = 12$	$12 \div 4 = 3$
		$4 \times 4 = 16$	$16 \div 4 = 4$
		$5 \times 4 = 20$	$20 \div 4 = 5$
		$6 \times 4 = 24$	$24 \div 4 = 6$
		$7 \times 4 = 28$	$28 \div 4 = 7$
		$8 \times 4 = 32$	$32 \div 4 = 8$
		$9 \times 4 = 36$	$36 \div 4 = 9$
		$10 \times 4 = 40$	$40 \div 4 = 10$

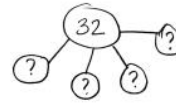
2. The baker packs 36 bran muffins in boxes of 4. Draw and label a tape diagram to find the number of boxes he packs.



$$36 \div 4 = 9$$

He packs 9 boxes of muffins.

3. The waitress organizes 32 glasses into 4 equal rows. How many glasses are in each row?



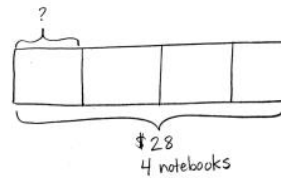
$$4 \times 8 = 32$$

$$32 \div 4 = 8$$

There are 8 glasses in each row.

4, 8, 12, 16, 20, 24, 28, 32

4. Janet paid \$28 for 4 notebooks. Each notebook costs the same amount. What is the cost of 2 notebooks?



$$28 \div 4 = 7$$

$$4 \times 7 = 28$$

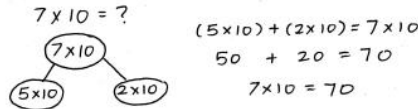
$$7 + 7 = 14$$

2 notebooks cost \$14.

# Lesson 18

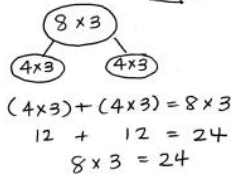
Objective: Apply the distributive property to decompose units.

5. There are 7 teams in the soccer tournament. 10 children play on each team. How many children are playing in the tournament?



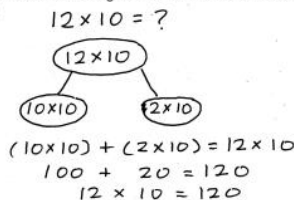
There are 70 children playing in the tournament.

6. What is the total number of sides on 8 triangles?  3 sides



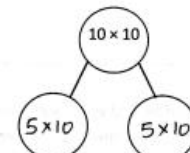
There are 24 sides altogether.

7. There are 12 rows of bottled drinks in the vending machine. Each row has 10 bottles. How many bottles are in the vending machine?



There are 120 bottles in the vending machine.

4.  $10 \times 10 = 100$



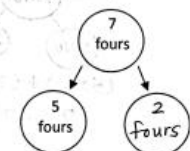
5 tens + 5 tens = 10 tens

$$(5 \times 10) + (5 \times 10) =$$

$$50 + 50 = 100$$

$$10 \times 10 = 100$$

2.  $7 \times 4 = 28$



5 fours + 2 fours = 7 fours

$$(5 \times 4) + (2 \times 4) =$$

$$20 + 8 = 28$$

$$7 \times 4 = 28$$

# Lesson 19

Objective: Apply the distributive property to decompose units.

d)  $32 \div 4 = 8$

$(20 \div 4) = 5$

$(12 \div 4) = 3$

$(32 \div 4) = (20 \div 4) + (12 \div 4)$

$= 5 + 3$

$= 8$

3. Nell draws the array below to find the answer to the division fact  $24 \div 2$ . Explain Nell's strategy.

Nell breaks apart 24 in 6 twos and 6 twos. 6 twos = 12. So she does  $12 \div 2$  for both parts and adds the answers to get  $24 \div 2$ .

$\leftarrow (12 \div 2) = 6$

$6 + 6 = 12$

$\leftarrow (12 \div 2) = 6$

$24 \div 2 = 12$

# Lesson 20

Objective: Solve two-step word problems involving multiplication and division and assess the reasonableness of answers.

3. Eighteen cups are equally packed into 6 boxes. 2 boxes of cups break. How many cups are unbroken?

$18 \div 6 = 3$

$4 \times 3 = 12 \text{ cups}$

12 cups are unbroken.

---

4. There are 25 blue balloons and 15 red balloons at a party. Five children are given an equal number of each color balloon. How many blue and red balloons does each child get?

blue  $25 \div 5 = 5$

red  $15 \div 5 = 3$

Each child gets 5 blue balloons and 3 red balloons.

---

5. Twenty-seven pears are packed in bags of 3. Five bags of pears are sold. How many bags of pears are left?

$27 \div 3 = 9$

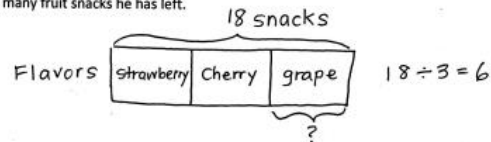
There are 9 bags of pears.

There are  $9 - 5 = 4$  bags left.

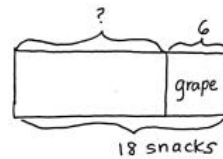
## Lesson 21

Objective: Solve two-step word problems involving all four operations and assess the reasonableness of answers.

3. Orlando buys a box of 18 fruit snacks. Each box comes with an equal amount of strawberry, cherry, and grape flavored snacks. He eats all of the grape flavored snacks first. Draw and label a tape diagram to find how many fruit snacks he has left.



$$18 \div 3 = 6$$

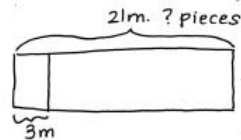


$$18 - 6 = 12$$

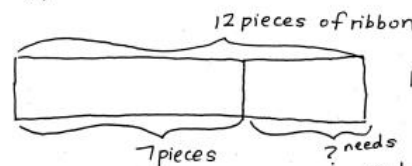
There are 12 fruit snacks left.

4. Eudora buys 21m of ribbon. She cuts the ribbon so that each piece measures 3m in length.

- How many pieces of ribbon does she cut?
- If Eudora needs a total of 12 pieces of ribbon, how many more pieces of ribbon does she need?



$$21 \div 3 = 7 \text{ pieces}$$



$$12 - 7 = 5 \text{ pieces}$$

Eudora needs 5 pieces of ribbon.

It's important to become fluent with multiplication and division facts. Quick 5-10 minute activities are essential for memorization. Here are some ways to assist your child with memorizing basic facts:

- Flash Cards
  - ◇ both you and your child should say the fact aloud
  - ◇ begin learning them in order
- Skip counting up and down. Try beginning at different starting points.
  - ◇ ie: 3, 6, 9, 12- 9, 6, 3      16, 20, 24, 28, 32-28, 24, 20, 16
- Have quick routine math talks in the car, store, and anywhere that seems appropriate.
- Computer Aides such as [xtramath.org](http://xtramath.org)